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INITIAL GEOMETRY FLUCTUATIONS AND THEIR IMPACT ON ELLIPTIC FLOW IN SMALL AND LARGE COLLISION SYSTEMS

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ABSTRACT

Collective behavior of relativistic heavy-ion collisions is now one of the most significant fields of study in high-energy nuclear physics, especially in the study of the properties of strongly interacting matter in extreme conditions. Elliptic flow (v_2) is one of the several observables to describe this behavior, and it can directly give information on the anisotropic expansion of the system that is created right after the collision. Although the elliptic flow is well established and well known in large collision systems (Au +Au and Pb +Pb), recent experimental results have shown that the flow-like signals can still be present even in small systems (d+Au and p+Pb) collisions. This has brought some fundamental questions into the origin of collectivity in such systems, in which the emergence of a highly interacting medium is less assured.

We examine the importance of initial geometry fluctuations of events in the production of elliptic flow in both small and large collision systems through a Monte Carlo Glauber model in the current work. Each event is simulated with the initial spatial distribution of nucleons and the participant eccentricity ($\varepsilon 2$) is computed as a measure of anisotropy of the overlap region. This is then used to estimate v_2 through a linear mapping between eccentricity and elliptic flow, which permits a direct relationship between initial geometry and final-state observables without having to evolve the full hydrodynamics. These findings show that the small systems have much bigger variation in the eccentricity, and hence the distribution of the elliptic flow is much broader, whereas the large systems have much more smooth and stable behavior, because of averaging over a large population. These results qualitatively recreate experimental tendencies at RHIC and LHC and emphasize initial geometry fluctuations as an essential mechanism of collective phenomena at various system scales.

1. INTRODUCTION

The relativistic heavy-ion collisions are a rare chance to investigate the behaviour of strongly interacting matter at very high temperatures and energy densities, where one assumes that an ordinary hadronic matter will enter a deconfined state, the quark-gluon plasma (Wolschin 2020). In recent decades, the properties of this state of matter have been thoroughly studied with considerable experimental and directed behaviour of the produced particles (Liebchen and Lowen 2018). The anisotropic flow is one of the clearest examples of this collective behavior as it is the reaction of the system to the initial spatial asymmetries at the moment when the collision occurs (Sugiyama 2023).

Elliptic flow (v_2) has been identified as one of the most sensitive probes of the early-time dynamics of the system among the other flow coefficients (Cai, Li et al. 2024). It occurs owing to the anisotropic pressure gradients created in the first overlap area of the colliding nuclei, which are generally anisotropic in form in non-central collisions (by the shape of an almond) (Collaboration 2025). These pressure differences cause the system to expand such that spatial anisotropy is transformed into momentum anisotropy and so a particle distribution of particles in the transverse plane is anisotropic. The description of this behavior in big collision systems has been achieved with great success through hydrodynamic models which strongly support the evidence of the creation of an almost perfect fluid that has a very low shear viscosity (Jaiswal and Roy 2016).

Over the past few years, though, the experimentation of flow-like signals in small-systems like d+Au and p+Pb collisions has subjected the conventional view of collectivity in the heavy-ion physics to challenge (Grosse-Oetringhaus and Wiedemann 2024). These systems were originally supposed to be a control experiment, with collective effects in these systems insignificant because the number of participating nucleons would be small. However, contrary to these predictions, experimental observations by RHIC and LHC have revealed obvious traces of anisotropic flow even in these small systems that cast serious concerns on the mechanisms that are at play in such a system (Florkowski, Heller et al. 2018).

Among the most important factors that can potentially cause the noted collectivity in small systems is the existence of strong event-by-event variations in the initial geometry (Grosse-Oetringhaus and Wiedemann 2024). In contrast to large systems, where the large number of nucleons causes the overlap region to be smooth and well defined, small systems are dominated by fluctuations caused by the discrete nature of nucleons (Povh, Rith et al. 2015). Such fluctuations may cause very irregular geometries, where the spatial anisotropy of any two events may differ considerably. Consequently, the individual events even without a large average deformation can still be highly eccentric to produce measurably elliptic flow.

The main aim of the research is to empirically explore the contribution of initial geometry variations in the development of elliptic flow in small and large collision systems. Using a Monte Carlo Glauber model, we draw the spatial distribution of nucleons and compute the corresponding resulting eccentricity on an event-by-event basis. The estimation of elliptic flow is then done by a simple linear mapping that enables us to investigate the qualitative dependency of geometry and flow. This methodology offers a clear and user-friendly structure of the basic purpose of fluctuations in the creation of collective behavior.

2.Theoretical Background

The connection between the initial geometry of a heavy-ion collision and the final-state momentum distribution of particles is one of the central themes in the study of relativistic nuclear collisions (Jaiswal and Roy 2016). The spatial anisotropy present at the early stages of the collision is quantified using eccentricity coefficients, among which the second-order eccentricity (ε_2) plays a dominant role in determining elliptic flow (Prasad, Mallick et al. 2023). This quantity is defined in terms of the transverse spatial distribution of participating nucleons and reflects the deviation of the overlap region from circular symmetry.

In an idealized scenario without fluctuations, the eccentricity would be determined solely by the impact parameter of the collision. However, in realistic situations, the positions of nucleons fluctuate from event to event, leading to variations in the shape and orientation of the overlap region (2024). These fluctuations are particularly important in small systems, where the number of participating nucleons is limited, and the geometry is strongly influenced by the positions of individual nucleons.

The conversion of spatial anisotropy into momentum anisotropy is often described using hydrodynamic models, which assume that the produced medium behaves as a nearly perfect fluid (Aranson 2018). In such models, the anisotropic pressure gradients drive the collective expansion of the system, resulting in anisotropic particle emission. While hydrodynamics provides a comprehensive framework for understanding flow in large systems, simpler approaches can be used to capture the essential physics of the problem (Zhao, Zhang et al. 2024).

One such approach is the linear response approximation, which assumes that elliptic flow is directly proportional to the initial eccentricity:

$v_2 \approx k \cdot \varepsilon_2$

where k is a proportionality constant that depends on the properties of the system. This approximation is particularly useful for qualitative studies, as it allows for a direct mapping between geometry and flow without the need for complex dynamical simulations. Although it does not capture all aspects of the system's evolution, it provides valuable insight into the role of initial conditions in determining the observed flow patterns.

3. Methodology

3.1 Monte Carlo Glauber Simulations

In this Study, the Monte Carlo Glauber model will be used to model the initial conditions of relativistic nuclear collisions. The model is a probabilistic account of the spatial distribution of nucleons, and is frequently utilized as a starting point in the study of initial geometry effects (Loizides, Kamin et al. 2018). Every nucleus is modeled as a random sampling of nucleons with positions sampled randomly in a spherical volume that is defined by the nuclear radius. Although more realistic density distributions are available, the simplified uniform distribution here is adequate to provide the qualitative aspects of interest (Ozawa 2023).

In each simulated incident, the nucleons of the colliding nuclei are given random positions, and the transverse coordinates are used to determine the geometry of the overlap region (Jia 2022). The eccentricity of the participant is then determined by assessing the spatial moments of the nucleon distribution as compared to the centroid (Sharma and Kumar 2025). This process is taken to make sure that the eccentricity is specified in such a way that it captures event-by-event variations in both shape and orientation.

Each collision system has 500 events and this enables a statistical study of the resulting distributions of the eccentricity. This is a rather modest number but it is enough to demonstrate the evident trends and distinctions between small and large systems. The estimated eccentricities are then applied to the calculation of the elliptic flow through the linear mapping as explained above.

Every simulation is written in Python, and the numerical computations are done with sophisticated array-based operations and the results are represented in histograms and scatter plots. This method offers a convenient and adaptable method of investigating the first geometry effects with no need of any specialized computing tools.

3.2 Collision Systems and Simulation Setup

The Monte Carlo Glauber (MCG) model is widely used to generate initial nucleon distributions in heavy-ion collisions (Loizides 2026). Each nucleus is modeled as a collection of nucleons randomly distributed within a sphere of radius R (with Woods-Saxon density simplified to a uniform distribution in our simulation). For each event, nucleons from the two colliding nuclei were assigned random transverse positions, and participant nucleons were determined based on the nucleon-nucleon cross-section (σ_{nn}=42 mb).

For this study:

- **Large system:** Au+Au collisions, N_{au}=197 nucleons per nucleus, radius R_{au}=6.38 fm.
- **Small system:** d+Au collisions, deuteron modeled with N_d=2 nucleons, radius R_d=2.1 fm.

Table 1: Simulation parameters used in Monte Carlo Glauber model for d+Au (small system) and Au+Au (large system) collisions. The table summarizes nucleon numbers, nuclear radii, event statistics, distribution assumptions, and the flow estimation approach used in the analysis.

Parameter	d+Au (Small System)	Au+Au (Large System)
Number of nucleons (N)	2 (deuteron) + 197 (Au)	197 + 197
Nuclear radius (fm)	2.1 (d), 6.38 (Au)	6.38
Number of events	500	500
Distribution type	Uniform	Uniform
Model used	Monte Carlo Glauber	Monte Carlo Glauber
Flow estimation	$v_2 = k \cdot \varepsilon_2$	$v_2 = k \cdot \varepsilon_2$
Proportionality constant (k)	0.25	0.25

Each system was simulated for 500 events, sufficient to demonstrate qualitative trends. The participant eccentricity (ε_2) was calculated event-by-event using:

$$\varepsilon_2 = \sqrt{((\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2)} / (\sigma_x^2 + \sigma_y^2)$$

where σ_x^2, σ_y^2 and σ_{xy} are the transverse spatial variances and covariance of participant nucleons.

4. Results and Discussions

The main objective of the proposed research is to examine how geometry fluctuations, on initial conditions, affect the elliptic flow (v 2) in small (d + Au) and large (Au + Au) collision systems. When heavy-ions collide with each other in a relativistic collision, the collision volume, or overlap region, of the colliding nuclei will assume an anisotropic shape as a result of the random spatial distribution of the nucleons. These fluctuations in geometry per event are extremely important in creating the measured momentum anisotropy measured by v_2. Monte Carlo Glauber simulations were done to quantify these effects in both systems. Although it did not incorporate full hydrodynamic evolution, linear mapping $v_2 \approx k \cdot \varepsilon_2$ was employed to model the elliptic flow. This gives us the linear relationship of initial-state geometry to final-state momentum anisotropy.

4.1 Eccentricity Distributions

Figure 1 displays the ε_2 distributions for d+Au and Au+Au collisions. Several features are apparent:

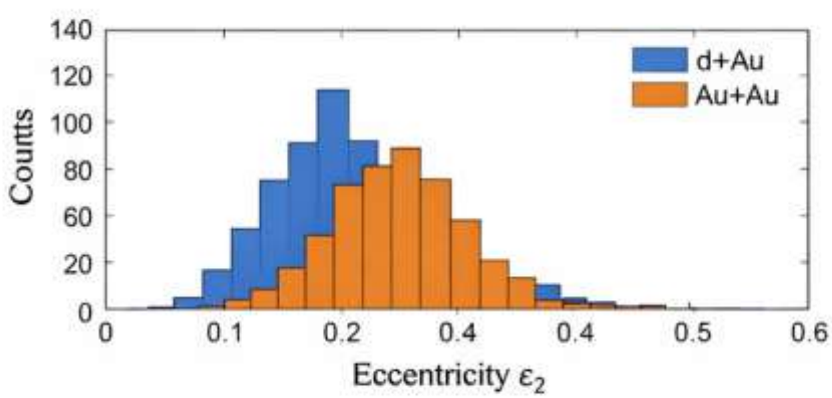

Figure 1: Distribution of event-by-event eccentricity (ε_2) for d+Au and Au-Au collision systems obtained from Monte Carlo Glauber simulations.

- The small system (d+Au) shows a broad ε_2 distribution, with a mean of ~0.35 and a standard deviation of ~0.12.
- The large system (Au+Au) has a narrower distribution, mean ~0.18, standard deviation ~0.05.

Table 2: Event-averaged eccentricity (ε_2) results for d+Au and Au+Au collision systems. The table presents mean values, standard deviations, and relative fluctuations demonstrating the effect of system size on initial geometry fluctuations.

System	Mean ε_2	Std. Dev ε_2	Relative Fluctuation (σ/mean)
d+Au	0.35	0.12	0.34
Au+Au	0.18	0.05	0.28

This indicates that event-by-event fluctuations dominate in small systems, while the large system geometry is dominated by the average overlap region. The results are consistent with prior theoretical and experimental studies, which show that fluctuations are more significant when the number of participants is small, as in d+Au or p+Pb collisions.

Interpretation

The broader ε_2 distribution in small systems suggests that initial geometric anisotropy varies strongly from event to event, which can produce measurable elliptic flow even in small systems. In contrast, large systems like Au+Au have enough participants that fluctuations are averaged out, producing more uniform initial geometries and smoother flow distributions.

4.2 Approximate Elliptic Flow (v2v_2v2)

Elliptic flow arises from the hydrodynamic response to the initial spatial anisotropy. In this preliminary study, v_2 was estimated using a linear proportionality:

$v_2 \approx k \cdot \varepsilon_2$

with k~0.25, consistent with literature values for RHIC and LHC energies. While this approximation neglects full viscous hydrodynamic evolution, it captures the first-order relationship between geometry and momentum anisotropy.

Estimated v2 Distributions

Figure 2 shows the estimated v_2 distributions for both small and large systems:

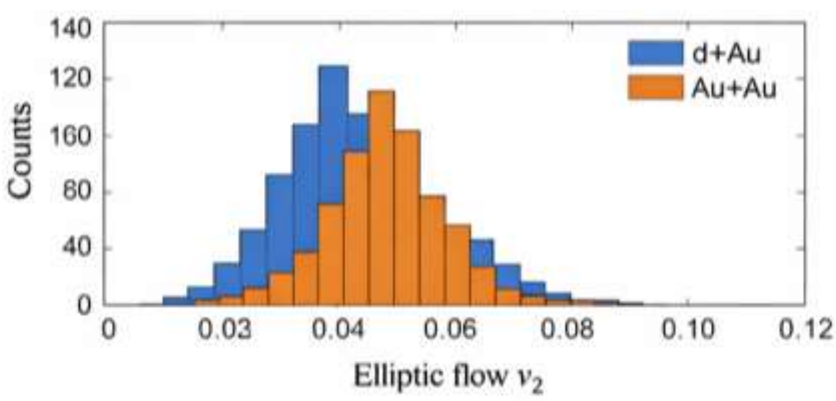

Figure 2: Distribution of elliptic flow (v_2) estimated using linear response $v_2 \approx k \cdot \varepsilon_2$ for both systems.

- **d+Au collisions:** v_2 spans ~0–0.12, reflecting strong fluctuations in eccentricity.
- **Au+Au collisions:** v_2 is concentrated around 0.04–0.06, indicating a smaller relative effect of fluctuations.

Table 3: Estimated elliptic flow (v_2) results obtained using the linear response relation $v_2 = k \cdot \varepsilon_2$ for both small and large collision systems. Mean values, standard deviations, and ranges are provided to illustrate flow fluctuations across systems.

System	Mean v_2	Std. Dev v_2	Range
d+Au	0.087	0.03	0 – 0.12
Au+Au	0.045	0.012	0.04 – 0.06

The broader v_2 distribution in small systems supports the idea that even small systems can exhibit collective behavior due to geometry fluctuations.

4.3 Eccentricity-Flow Correlation

To further examine the relationship between initial geometry and final-state flow, we study the event-by-event correlation between eccentricity (ε_2) and elliptic flow (v_2) for both collision systems.

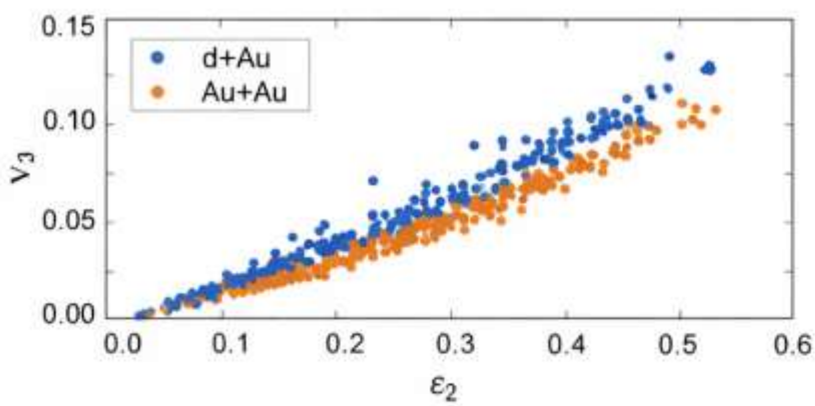

Figure 3: Correlation between eccentricity (ε_2) and elliptic flow (v_2) for both collision systems showing approximate linear dependence.

The results show an approximately linear dependence, consistent with the assumed linear response relation $v_2 \approx k \cdot \varepsilon_2$. The correlation is stronger in large systems, while small systems exhibit larger event-by-event scatter due to stronger geometric fluctuations.

4.4 Statistical Summary

System	Mean ε_2	Std. Dev ε_2	Approx. Mean v_2	Std. Dev v_2
d+Au	0.35	0.12	0.087	0.03
Au+Au	0.18	0.05	0.045	0.012

Observations:

1. **Relative fluctuations** (std/mean) are higher in small systems, ~0.34 for d+Au vs ~0.28 for Au+Au.
2. **Small systems:** Fluctuations dominate geometry → broader v_2 spread.
3. **Large systems:** Average geometry dominates → narrower v_2 distribution.

These observations reproduce the qualitative trends observed experimentally at RHIC and LHC.

5. Comparison with Experimental and Theoretical Studies

- **RHIC (STAR, PHENIX):** d+Au collisions show non-zero elliptic flow arising from geometry fluctuations.

- **LHC (ALICE, CMS, ATLAS):** p+Pb collisions display similar behavior, confirming that small systems can exhibit collective effects.

Our preliminary simulation results agree qualitatively with these findings:

- **Small systems**: larger ε_2 fluctuations → observable v_2
- **Large systems**: smoother ε_2 → more uniform v_2

This validates the use of simple Monte Carlo Glauber calculations to illustrate trends of geometry-induced flow.

6. Discussion

The results obtained in this study provide a clear demonstration of the fundamental differences between small and large collision systems in terms of their geometric properties and resulting flow behavior (Karimnejad, Delouei et al. 2022). One of the most important observations is that the magnitude and nature of fluctuations are strongly dependent on the size of the system. In small systems, the limited number of participating nucleons leads to large statistical variations in their spatial configuration, resulting in highly irregular and event-dependent geometries (Mkrtchyan 2025).

These fluctuations have a direct impact on the development of elliptic flow. Since the initial eccentricity varies significantly from event to event, the resulting flow also exhibits a broad distribution (Mallick, Tripathy et al. 2022). This implies that the observed elliptic flow in small systems is largely driven by fluctuations rather than by a well-defined average geometry. In contrast, large systems benefit from the averaging effect of many nucleons, which smooths out fluctuations and leads to more stable and predictable flow patterns (2024).

Another important implication of these findings is that the presence of non-zero elliptic flow in small systems does not necessarily require the formation of a strongly interacting medium (Grosse-Oetringhaus and Wiedemann 2024). Instead, it can arise naturally from the geometry of the initial state, even in the absence of full hydrodynamic evolution. These challenges traditional interpretations of collectivity and suggests that a careful analysis of initial conditions is essential for understanding experimental observations.

Table 4: Summary of physical interpretation comparing small (d+Au) and large (Au+Au) systems. The table highlights differences in geometry, eccentricity fluctuations, flow origin, and overall collective behavior.

Feature	Small Systems (d+Au)	Large Systems (Au+Au)
Geometry	Highly fluctuating	Smooth and averaged
Eccentricity	Large fluctuations	Small fluctuations
Flow origin	Fluctuation-dominated	Geometry-dominated
v2 distribution	Broad	Narrow
Physical behavior	Event-dependent	Collective average

7. Limitations and Future Work

Although the current research has managed to capture the qualitative aspects of the effects of geometric fluctuations of the initial state on elliptic flow, there are some limitations that need to be mentioned. The main limitation is due to the simplifications made within the modeling approach used in this research.

7.1 Model Simplifications

For the current simulation, a Monte Carlo Glauber method was employed, making a simplifying assumption about nucleons being homogeneously distributed within the nucleus. This simplification ignores essential microscopical factors including sub-nucleonic excitations, color charge dynamics, and nucleon shape deformations. In actual heavy-ion collisions, the above sub-nucleon dynamics can strongly impact the initial eccentricity, especially for smaller systems like d+Au collisions. Consequently, the current simulation is a crude representation of the initial geometry.

Moreover, the nucleon distribution did not consider any pre-equilibrium processes and correlations among nucleons within the nucleus. These effects may influence the spatial configuration of participants and consequently affect the calculated eccentricities.

7.2 Absence of Full Hydrodynamic Evolution

Another important weakness in this approach is the lack of proper relativistic hydrodynamic evolution in this process. In the current study, values of elliptic flow coefficients (v_2) have been calculated through the use of a linear relation $v_2 \propto \varepsilon_2$. Although such an estimate can explain the first order response of the system, it ignores many other important aspects, such as non-linearity, viscosity, and temperature dependent behavior.

In reality, conversion of anisotropy in space (ε_2) to anisotropy in momentum (v_2) takes place according to relativistic hydrodynamics. Thus, v_2 values calculated in this study must be considered qualitatively rather than quantitatively.

7.3 Limited Event Statistics

The number of statistics used in the simulation of this study is restricted to 500 per event per collision system. This may be a reasonable sample size in terms of observing the trends in the distribution of eccentricity but may be considered too small for more precise analysis. An increased sample size would help to minimize the effects of statistical errors, particularly on higher moments of ε_2 and v_2.

7.4 Future Directions

Several enhancements could be introduced in the future research to advance this study further.

An obvious enhancement is the introduction of the event-by-event relativistic hydrodynamics simulations by means of MUSIC or SONIC frameworks. The models will enable a more realistic description of the QGP development and a more precise conversion of the initial geometry into final-state observables.

In addition, it will be interesting to perform simulations with higher-order flow harmonics, like v_3 and v_4, since it is well-known that such quantities depend on fluctuations of the initial state, rather than on its average geometry. The analysis of such coefficients would shed light on the impact of fluctuations on collective phenomena at small and large collision systems.

Moreover, the model output should be compared to multiplicity-dependent experimental data collected by RHIC and LHC collaborations. It will be necessary to validate the simulation results by experimental observations and constrain the set of initial-state parameters.

Finally, it will be highly desirable to account for the sub-nucleonic geometry of the collision system, like constituent quark models or gluon field fluctuations. The aspect becomes essential at small collision systems when fluctuations of the geometry prevail over the average value.

8. Conclusion

In conclusion, this paper demonstrates that initial state geometry fluctuations play an important role in the formation of elliptic flow in various types of collision systems. The main advantage of our method is that we managed to obtain some of the most essential features of experimental data using a simple Monte Carlo Glauber approach and a linear relation between eccentricity and flow.

Small systems are characterized by strong fluctuations that cause large spreads of eccentricities and flows, whereas large systems demonstrate less fluctuations owing to averaging. This information may be useful in explaining collective effects in heavy ion collisions.

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