INTERNATIONAL JOURNAL OF SOCIAL SCIENCES BULLETIN
ISSN: (E) 3007-1917 (P) 3007-1909

Volume 4, Issue 3, 2026

ENHANCING FOUNDATIONAL NUMERACY THROUGH THE CONCRETE–PICTORIAL–ABSTRACT (CPA) APPROACH: EVIDENCE FROM A QUASI-EXPERIMENTAL STUDY IN PAKISTANI PRIMARY SCHOOLS

Tahmina Bano

Montessori Directress, Early Childhood Education Trainer & M Phil Education Research Scholar, College of Education FLAHS, Ziauddin University, Karachi, Pakistan

rizvi.tahmina@gmail.com

Corresponding Author: *
Tahmina Bano

DOI: https://doi.org/10.5281/zenodo.21847032

Received	Accepted	Published
03 February2026	15 March 2026	31 March 2026

ABSTRACT

Foundational numeracy skills are essential for children to build on for a successful future in school and life. In Pakistan, many primary school-age children struggle in school because of the teaching methods that focus on rote learning, as opposed to the teaching methods that focus on understanding. This study focuses on the Concrete-Pictorial-Abstract model (C-P-A) and attempts to articulate the real understanding of foundational numeracy skills of primary school children in Pakistan. A quasi-experimental research design was used and comprised an experimental group that was taught mathematics using the C-P-A model and a control group that was taught mathematics using the conventional model of teaching. Data were obtained using a pre-test and post-test numeracy achievement assessments. The results are anticipated to show a noteworthy difference in number sense, reasoning, and problem-solving skills of students taught using the C-P-A model of teaching as opposed to students taught using the conventional model of teaching. The study attempts to show the importance of teaching young children mathematics using a combination of real learning materials, pictures and drawings, and forms of mathematics that they can symbolize. The study attempts to improve early mathematics teaching in primary schools of Pakistan.

Keywords: *Foundational Numeracy, Concrete–Pictorial–Abstract (CPA) Approach, Primary Mathematics Education, Quasi-Experimental Study, Mathematics Achievement, Early Grade Learning, Pakistani Primary Schools.*

INTRODUCTION

Foundational numeracy is the understanding and use of mathematics needed to solve real world problems, especially, problems that one may encounter during childhood. Children learn to use foundational numeracy skills during childhood, and the lack of these skills affects the successful teaching and learning of other subjects. Many mathematics classrooms in developing countries especially in developing countries like Pakistan offer little support to the development of foundational numeracy. These foundations include a sense of and the ability to use numbers, knowledge and skills to solve mathematical problems, and the use of active exploration to learn mathematics. The use of these foundations and active participation by learners is essential for the successful teaching and learning of

mathematics. Clements and Sarama (2018) emphasized the development of a sense of numeracy through creative and age appropriate learning activities, while the World Bank (2022) stressed the improvement of teaching and learning of numeracy as an essential and basic requirement for the overall improvement of teaching and learning in developing countries.

The Concrete–Pictorial–Abstract (CPA) approach integrates concrete experiences, visual aids, and symbolic representations to build students' conceptual understanding of mathematics. The approach involves the use of concrete materials, pictorial representations, and abstraction of numbers and operations, respectively. With this approach, learners are actively engaged and motivated to build conceptual understanding of mathematics. According to Carbonneau et al. (2018), manipulatives positively impact learners' conceptual understanding of mathematics when they are carefully aligned with teaching processes. Fyfe et al. (2019) stated that thinking of mathematics in the abstract is built progressively and positively impacts learners' logical thinking. The findings of the study conducted by Putri et al. (2020), indicated that students' mathematical reasoning was positively impacted by the CPA approach, as opposed to the conventional teaching methods. Furthermore, Moyer-Packenham et al. (2021), also stated that visual aids are very important in the development of young learners' logical thinking and creativity in the solving of mathematical problems. Finally, Lai and Yeo (2021), stated that the CPA approach positively impacts students' engagement, and confidence in the learning of mathematics, as it reduces the challenges presented by the abstraction of mathematical concepts.

Though more countries endorse CPA-based mathematics instruction, few studies examine the practice in Pakistan's primary schools. Pakistan's education system struggles to build students' foundational mathematics skills. This creates gaps in classroom learning that evidence-based teaching strategies may fill. UNICEF (2021) argued that teaching innovations that close learning gaps are crucial for foundational skills. Azzumar and Juandi (2023) reported positive impacts of CPA-based instruction on mathematics across varied educational contexts, provided the value of context adaptation. Aguilar and Becerra (2022) considered multiple representations of CPA to aid the development of students' mathematical understanding and relate varied knowledge. Furthermore, the Ministry of Federal Education and Professional Training (2020) establishes that improving the foundational learning of the student population is the most important priority in Pakistan's education. Given this, the aim of this study is to examine the benefits of Concrete-Pictorial-Abstract to reinforce foundational numeracy among primary students in Pakistan, and to provide evidence-based practice recommendations for mathematics instruction.

Background of the Study

Foundational numeracy is critical during early education. Young learners begin to build the knowledge they need to identify and manipulate numbers. These skills are critical in reasoning and solving problems. Early education also solidifies skills learners will need later to pursue higher levels of learning and apply quantitative skills to their studies and everyday activities. Learning maths in primary school requires teachers to adopt techniques that go beyond teaching by rote and instead promote the learners developing an understanding of the maths concept through valuable learning experiences. Clements and Sarama (2018) asserted that teaching young learners' advanced maths concepts enables the learners to construct strong foundations of maths which relates to their future advanced maths skills. Unfortunately, outside developed nations, many learners find learning maths a challenge. Effective teaching, appropriate resources, and sufficient opportunities to interact with the maths concepts are critical in overcoming the maths learning challenges. According to the World Bank (2022), in developing nations, including many low- and

mid-income countries, closing learning gaps is a key global educational focus, improving early maths learning and numeracy. In Pakistan, the state of a pupil's basic arithmetic skills is an indicator of the poor state of education in the country. Primary school mathematics teaching is often dominated by rote, formulaic, and teacher-centric methods of teaching. Such methods are likely to stifle a pupil's ability to see the relevance of mathematics to the real world, and to develop a practical relationship between the two. UNICEF (2021) argued that developing basic skills in a pupil entails changing the teacher-centric methods to those that stimulate the pupils' cognitive skills and participation. The Concrete–Pictorial–Abstract (CPA) establish a pathway a pupil can follow to develop their understanding of Mathematics. It starts with the Concrete phase, where a pupil is provided with real, physical teaching aids. This helps the pupil to form mental images of the mathematical concepts. The aids are then gradually removed until the pupil is left only with abstract concepts. A quasi-experimental study on the effectiveness of CPA-based teaching in Pakistani primary schools would provide much needed data to strengthen the teaching of primary school mathematics.

Foundational Numeracy and Early Mathematics Learning

Foundational numeracy encompasses the first school-age numeracy skills that are critical to a child's ability to learn advanced math. Children learn to recognize numbers and patterns, quantitative comparisons, and basic math operations and reasoning. However, children do not just learn math skills; they learn to apply math to real-world situations and learn to think critically. Mullis et al. (2020) posit that children with a solid foundation of early skills develop the confidence to understand math and, as a result, perform better in all subsequent math classes.

To learn math, children must understand the concepts. There are many strategies that teachers can use to help children understand math concepts and help them become better problem solvers. Fyfe et al. (2019) describe that helping children understand the idea that there can be math in the real world and that not everything is concrete can help them think beyond just the situation and use the math they know. Because of this, when developing early math skills, teaching the child how to understand the math rather than just memorizing it is critical.

Challenges of Numeracy Development in Pakistani Primary Schools

Mathematics learning outcomes in Pakistan face many obstacles despite numeracy's central role when considering primary education. Traditional teaching methods see teachers explain mathematics procedures to students who in turn memorize the rules and formulas. Opportunities for students to do and visualize mathematics are curtailed. Related to this is the unavailability of teaching aids, large student populations, and inadequate teacher training.

A large obstacle is the difference between the expected curriculum and what happens in the classroom. Teaching documents endorse learning through the acquisition of skills. However, teachers do not receive support to adopt student-centered strategies. In the Ministry of Federal Education and Professional Training's (2020) report, it is mentioned that to improve Pakistan's education system, the challenges of untaught basic skills and poor quality teaching must be addressed. Innovative strategies are called for to enable students to learn the core mathematics concepts and to support teachers.

The Concrete–Pictorial–Abstract (CPA) Approach in Mathematics Education

Concrete – Pictorial – Abstract (CPA) is a technique designed to develop a student's understanding of an idea by framing it in successive forms of representation. The Concrete stage involves students using physical objects to explore an idea. These objects could be counters, blocks, or anything from a student's environment. The next stage involves students drawing or making diagrams of the idea. Finally, the last stage involves the most abstract representation of the idea where the students

construct and solve a mathematical statement using symbols and numbers.

The first stage of the CPA approach suggests students first need to experience an idea in the real world before they are able to work with abstract symbols. The Pictorial and Concrete stages of the CPA approach provide students with tools that improve their mathematical reasoning. Carbonneau et al. (2018) suggest manipulatives are most effective when connected to an idea with instructional purpose and explanation of the idea at a conceptual level. Similarly, Putri et al. (2020) found significant improvements in elementary students' mathematical reasoning when a CPA approach to teaching was used, as compared to Traditional Teaching Methods.

Rationale of the Study

The aim of this study is to build upon what is already known regarding the situation regarding numeracy among primary school children in Pakistan, to identify the possible barriers to CPA based instruction, and to create opportunities for the early incorporation of CPA instruction at this stage of school. Most of the teaching methods used for instruction in Pakistan, like many other countries, do not help children to develop their understanding of mathematics and reasoning skills. Since primary math skills will determine the level of success the child will have at school, the task of teaching children the ways to handle math and develop the thinking skills that go with it, becomes even more critical.

One of the methods that can be used to teach with action, is the CPA approach (Concrete, Pictorial and Abstract). This approach is limited inPrimary Schools in Pakistan, and thus cannot be used in many other schools in the region. This study will provide the much needed evidence to demonstrate precisely what role and value educational instruction based on the CPA method will provide for the enhancement of early numeracy skills for children.

Research Gap

Although prior studies have shown that the CPA approach positively influences mathematical learning, most studies have been conducted in other countries. Limited empirical studies are available on CPA-based instruction in Pakistani primary schools. Also, very few studies have explored the effectiveness of CPA using a quasi-experimental design and focusing on the development of basic numeracy. Thus, this study attempts to fill this gap and determine if the use of CPA-based teaching can enhance numeracy achievement of primary school children in Pakistan.

Research Objectives

1. To examine the effect of the Concrete–Pictorial–Abstract approach on students' foundational numeracy skills.
2. To compare the numeracy achievement of students taught through CPA-based instruction and conventional teaching methods.
3. To determine the impact of CPA instruction on students' mathematical reasoning and problem-solving abilities.

Research Questions

1. Does the Concrete–Pictorial–Abstract approach significantly improve foundational numeracy skills among primary school students?
2. Is there a significant difference between students taught through CPA instruction and traditional teaching methods?
3. How does CPA-based instruction influence students' mathematical reasoning and conceptual understanding?

Significance of the Study

The objective of this research is to add to the body's work on mathematics education by providing evidence to support the effectiveness of the CPA approach on primary students' foundational numeracy. This evidence may influence teachers to implement more effective teaching strategies based on students' understanding and engagement. For curriculum developers and policy makers, the evidence-based

methodology of this research study may assist in developing primary mathematics curriculum. The evidence may encourage research of advanced mathematics pedagogy in Pakistan and other educationally developing countries.

LITERATURE REVIEW

The early stages of education provide children with the skills that help them understand the fundamentals of mathematics, such as numbers, calculations, and how to solve math problems. Foundational numeracy includes skills such as counting, identifying and comparing numbers, and solving math problems. It encompasses logic and reasoning, as well as an understanding of patterns and a developed sense of math. The early stages of education are critical for the development of numeracy skills. A strong foundation of early numeracy skills is essential for children to master math and use numeracy skills throughout their lives. Reliable early math skills help children excel in all areas of education. Clements et al. (2019) stated that early education offers children the skills to develop math-related skills of logic and reason and the ability to solve problems. Similarly, Duncan and Magnuson (2018) stated that early math skills directly correlate with future success in school and therefore strong math teaching in the early years is extremely important.

Conceptual math skills must be prioritized over procedural skills in order to develop foundational math skills. Young children must participate in activities that are meaningful to them in order to successfully build math skills. Griffiths et al. (2020) stated that early education of math should allow children to build skills and understand concepts of math numerically through many different representations and solve many different math-related problems. Foundational numeracy skills can be improved by altering the teaching methods and strategies to promote understanding over memorization and participation.

Importance of Concrete Learning Experiences in Mathematics

Young learners use concrete learning experiences to help understand abstract ideas, like those in mathematics. Learning primary mathematics is full of difficulties. Learning abstract ideas like symbols and procedures are often learned when tangible experiences are unavailable. Physical objects, like counters, sticks, or even real-life objects, help students learn mathematics by providing physical ways to learn ideas and concepts. As noted by Carbonneau et al. (2018), manipulatives help learn mathematics when learning experiences are physical and ideas are mathematical.

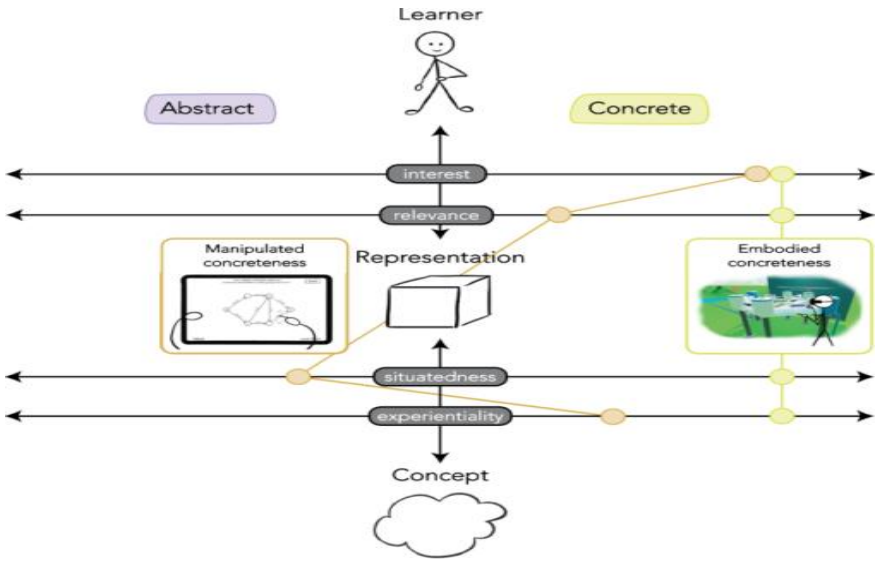

Fig 2.1 Importance of Concrete Learning Experiences in Mathematics

Concrete materials help students understand the concepts of mathematics. Student conceptualization of ideas and relationships of mathematics is possible because students are able to use physical manipulatives before they are required to use mathematics symbols. As noted by Fyfe et al. (2019), students learn and understand mathematics best when concrete learning experiences are integrated and connected to abstract ideas of mathematics. Concrete learning ideas and experiences are best used to learn primary mathematics.

Role of Visual Representation in Mathematical Understanding

Visual representation is a key ingredient that cultivates a thorough understanding among students during the learning of mathematics especially in the understanding of the relationships of various mathematical concepts. Through diagrams and different representations, learners are able to make sense of the information available and organize it in a structured manner. Visual representation is highly advantageous when working with younger students, whose learning involves breaking the relationships of the concepts from the abstract and creating connections with the symbols. Moyer-Packenham et al. (2021) stated that visual representation encourages students' mathematical reasoning, as it allows them to understand and appreciate the relationships and reasoning of the concepts which are hard to appreciate when working with the mathematics symbols.

Traditionally, classroom discussions reinforce learners' ability to articulate mathematical reasoning and the solutions to mathematical problems. Visual representation encourages students to move beyond the memorization of the mathematical concepts. This is achieved in understanding the reasoning as to why the mathematical processes are structured in a particular manner. According to Lesh et al. (2018), receptive learners are able to create links between their concrete experiences and the visual representation of the mathematical models with reasoning when different representations are used.

Transition from Concrete to Abstract Mathematical Thinking

Properly equipping students to transition from concrete to abstract thinking is an important aspect of math instruction. Customers often need support as they physically move from handling objects to writing symbols to executing formal processes. Initially developing instructional frameworks that scaffolds advancement is critical in developing self-confidence and self-sufficiency of learners. The Concrete-Pictorial-Abstract (CPA) model offers structured progress and built self-confidence and self-sufficiency of learners. The CPA model allows for self-confidence, self-sufficiency and informally describes the stages of mathematical representation process.

Kaur, 2019, cites the Bruner stages of representation as an example where learning can be developed in three areas: Enactive, iconic and symbolic representation. The CPA model of instruction is seen where students first act physically about abstract things, then represent abstract things visually, and at last represent abstract things symbolically. This gradual transition helps students develop and refine abstract thinking along the way.

The Concrete–Pictorial–Abstract (CPA) Approach in Mathematics Education

In the Concrete-Pictorial-Abstract method, the objective is to create an interconnected web of different representations for students to help improve their ability to learn and carryout mathematical tasks. In the Concrete stage, students use solid/manipulatable objects to gain an understanding of a given mathematical concept. In the Pictorial stage, students create drawings or utilize diagrams and other visual models to aid in their understanding of a mathematical concept. In the Abstract stage, students solve a given mathematical problem with the use of symbols and mathematical operations. The stages of the Concrete-Pictorial-Abstract method help learners build their understanding

of mathematical concepts before they are required to use a more abstract form of mathematical concepts.

An emerging body of evidence supports the effectiveness of CPA-based instruction in primary-level mathematics. Putri et al. (2020) claim that children taught by CPA methods performed better with mathematical reasoning when compared to the children that received the conventional method of teaching. In addition, Azzumar and Juandi (2023) stated that the CPA method of teaching positively affected the understanding and problem-solving skills of the learners in mathematics, as well as the students' overall achievement in the different settings of education.

CPA Approach and Students' Numeracy Achievement

The use of CPA and its influence on numeracy achievements have been reviewed in different educational contexts. Evidence suggests that when students utilize CPA, they are able to achieve advanced number sense, enhanced calculating and reasoning skills, and improved understanding of mathematics, as they are stimulated to link real-life experiences with the use of symbols. Students that engage with CPA tend to articulate their understanding in the absence of formal instructions, as they are able to interpret and visualize the concepts of mathematics with ease.

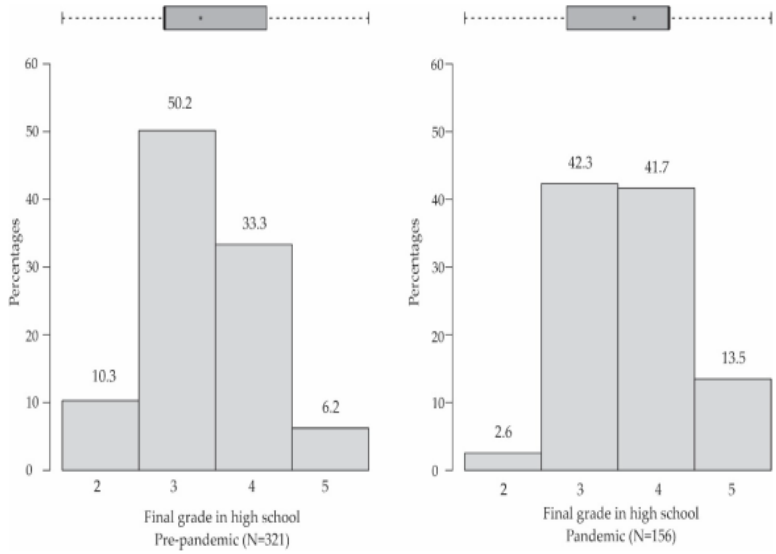

Fig 2.2 CPA Approach and Students' Numeracy Achievement

Lai and Yeo (2021) stated that using CPA instruction, primary school students' problem-solving skills, as well as their self-confidence pertaining to their learning of mathematics, showed marked improvement. Further, Aguilar and Becerra (2022) articulated that the use of multiple representations, concrete and pictorial, facilitates the construction of more profound and intricate links to the field of mathematics. From these studies, CPA opens the pathways to solving challenges that are usually seen by early-stage learners of mathematics.

CPA-Based Mathematics Instruction in Developing Countries

Limited teaching resources, poor teacher training, and rote learning converge to create a poor state of mathematically based education in most developing countries, but locally available materials and understanding-based active learning created by the CPA approach have the potential to provide realistic options for developing countries and build understanding of mathematics. UNICEF (2021) stressed that contextually focused teaching methods that assist children in their cognitive development are a

primary necessity for strengthening elementary learning.

In the numerically challenged developing country of Pakistan, many primary classrooms depend on teacher-centered mathematics instruction. CPA strategies may be able to address the challenge that most students of Pakistan face in the development of numeracy. However, the adaptation of CPA strategies to the Pakistani context still needs further research. There is a need for more quasi-experimental studies to assess the role of CPA in strengthening basic numeracy.

RESEARCH METHODOLOGY

This study used a non-experimental research design to assess how successful the Concrete–Pictorial–Abstract (CPA) methods are in developing fundamental numeracy skills with primary learners. A non-experimental design was used since random assignment was not possible for the participants within the school settings available. The study used a pre-test and post-test design with control group design and the control group received mathematics instruction with the CPA approach and the other group with traditional instruction methods was used. The design gave the researchers the opportunity to assess the changes in the students' achievement in numeracy after the intervention.

Table 1: Demographic Characteristics of Participants

Characteristics	Experimental Group (n=30)	Control Group (n=30)	Total (N=60)
Male Students	16 (53.3%)	15 (50.0%)	31 (51.7%)
Female Students	14 (46.7%)	15 (50.0%)	29 (48.3%)
Grade Level	Grade 2	Grade 2	Grade 2
Mean Age (Years)	7.8	7.9	7.85

Research Approach

The study implemented a quantitative approach to consider the effect of the CPA intervention. This required data collection in the form of students' achievement scores. The quantitative approach was suitable for evaluating whether notable changes occurred between the experimental and control groups following the instructional intervention.

Research Setting

This research was carried out in multiple public primary schools in Pakistan. Primary school level education is where students learn basic mathematical concepts. Many public primary school students face difficulties with basic skills. This made the schools a good context to study the actual practice and efficacy of CPA-oriented mathematicalinstruction.

Table 2: Sample Distribution According to Groups

Group	Teaching Method	Number of Students	Percentage
Experimental Group	CPA-Based Instruction	30	50%
Control Group	Conventional Instruction	30	50%
Total	—	60	100%

Population of the Study

The population of this study was primary school students at early grades of Pakistani schools. The target population consisted of students who were developing early numeracy skills. This included students learning such basic concepts as counting, number identification, addition, subtraction, and even basic problem-solving.

Sample and Sampling Technique

Elementary school students were chosen in through purposive sampling. From there,

participants were randomly designated to two groups: experimental and control. Participants in the experimental group received mathematics instruction based on the CPA approach. Those in the control group were taught the corresponding mathematics instruction using conventional methods. The defining factors for the selection of participants to include accessibility, cooperative schools, and the convenience of the participants to the researcher.

Table 3: Description of Research Instruments

Instrument	Purpose	Number of Items	Measurement Area
Foundational Numeracy Achievement Test	Measure students' numeracy skills	25	Number sense, calculations, problem-solving
Classroom Observation Checklist	Observe CPA implementation	15	Concrete, pictorial, abstract activities
Student Performance Record	Monitor learning progress	–	Achievement improvement

Research Instruments

Data collection utilized a Foundational Numeracy Achievement Test designed in line with primary-level mathematically curricular goals. The test assessed students' comprehension of fundamental numeracy, number operations, and the ability to solve a mathematical problem. The test items were designed to assess students' conceptual and procedural knowledge prior to and subsequent to the intervention.

Table 4: Structure of CPA-Based Instructional Intervention

Instructional Stage	Learning Activities	Teaching Materials	Expected Learning Outcome
Concrete Stage	Students manipulate objects and solve numerical tasks	Blocks, counters, number cards	Understanding basic concepts
Pictorial Stage	Students use drawings and visual models	Pictures, diagrams, charts	Developing visual understanding
Abstract Stage	Students solve symbolic mathematical problems	Numbers, equations	Applying mathematical procedures

Classroom observation checklists were used for assessment of the intervention for the adoption of CPA-based teaching practices. The observation checklist focused on the use of manipulatives, representation, student activity, and movement through the concrete, pictorial, and abstract phases.

Validity and Reliability of Instruments

The achievement test items underwent an expert review for content validity. Suggestions from the experts were integrated into the items for the achievement test. Test item review included the measurement and analysis of the item and test reliability. Statistically analysis of test results ensured consistent measurements of the participants' numeracy skills.

CPA-Based Instructional Intervention

Using the Concrete–Pictorial–Abstract method, the intervention followed three progressive stages in each math lesson for the experimental group.

In the concrete stage, physical learning resources (counters, blocks, and other manipulatives) help students engage and learn about mathematical concepts. In the pictorial stage, learners represent ideas and concepts through drawings. Students use the abstract stage to write and solve problems using the appropriate symbols.

INTERNATIONAL JOURNAL OF SOCIAL SCIENCES BULLETIN
ISSN: (E) 3007-1917 (P) 3007-1909
Volume 4, Issue 3, 2026

The talking intervention was based on the teaching of the CPA approach within a specified instructional period. Lessons were based on the learners' grade and needs. The teacher supported and guided the learners to abstract concepts and ideas of mathematics.

Data Collection Procedure

Data collection occurred across three distinct steps. First, a pre-test was done to assess the baseline numeracy skills of both groups. This happened prior to the intervention. Second, the intervention, which was CPA-based, was done on the experimental group, while the control group partook in the regular mathematics class. In the last step, both groups were given a post-test to assess the growth in students' numeracy achievement post intervention.

All participants were informed of the study, and data were collected with confidentiality and ethics of research in mind.

Data Analysis Techniques

Descriptive and inferential statistics were applied after data collection and included analysis of sample means and standard deviations for descriptive statistics and for inferential statistics, analysis of the extent of the difference between the control and experimental groups.

For the comparison of achievement differences between the control and experimental groups, independent samples t-test was used, while evaluation of the extent of the improvement of the experimental group through the intervention was measured by the difference between the control and experimental groups' pre-test and post-test through a paired samples t-test. The size of the effect of the CPA intervention was evaluated to determine the significance of the improvement of the participants' foundational numeracy development.

Ethical Considerations

Pursuant to ethical guidelines, the necessary permission was sought from the appropriate school administration prior to data collection. Students were not obliged to participate, and their information was kept private. Data collected were only used for the stated purpose of the research, and, to the best of my ability, the reporting of the findings was done without harming or placing the participants at any risk.

RESULTS ANALYSIS

This section includes the study results from pre-test and post-test data. The results will show the effectiveness of the Concrete-Pictorial-Abstract (CPA) method in the enhancement of primary school student foundational numeracy skills. Performance results of students from the experimental group who received instruction based on CPA, and from the control group who received instruction based on the traditional method, were analyzed statistically.

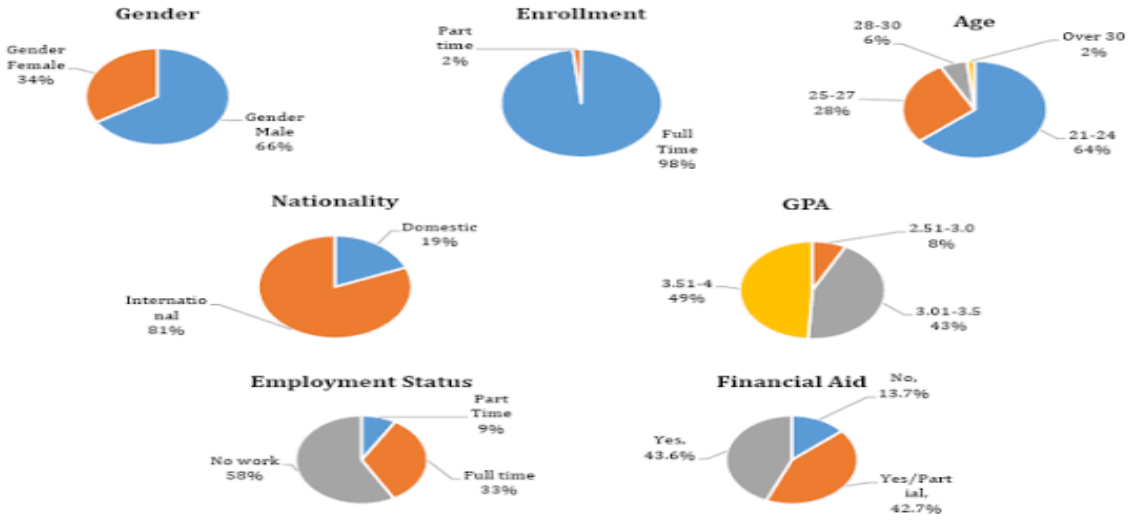

Table 5: Demographic Profile of Students

Variable	Frequency	Percentage
Male	31	51.7%
Female	29	48.3%
Total	60	100%

Demographic Characteristics of Participants

The quasi-experimental intervention consisted of students from selected primary schools in Pakistan. These students were assigned to either the experimental group or the control group. The students in both groups had the same educational background and the same level of learning. The sample for the study was confirmed to be appropriate for the study by analyzing the distribution of students by their grade level and by their group and by the demographic characteristics of the students.

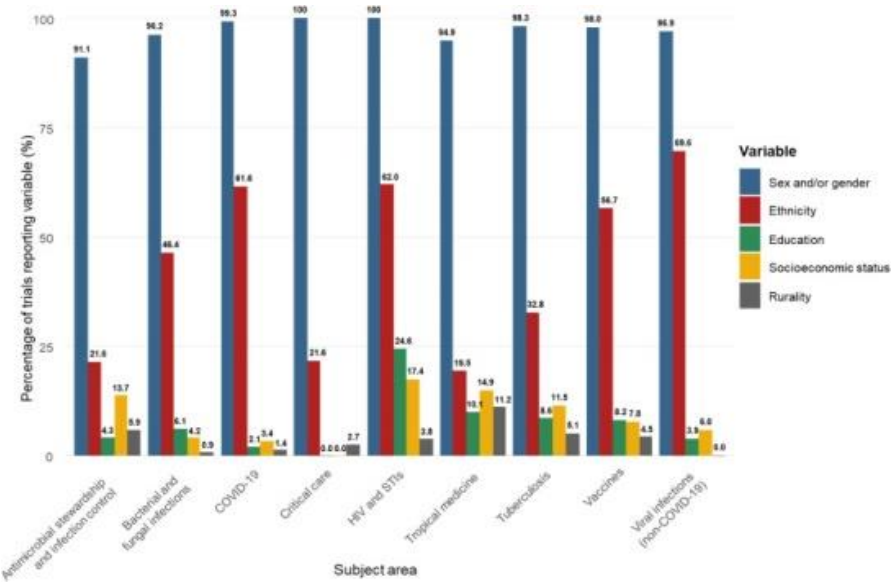

Fig 4.1 Demographic Characteristics of Participants

Table 6: Pre-Test Scores of Experimental and Control Groups

Group	N	Mean Score	Standard Deviation	t-value	p-value
Experimental Group	30	14.26	3.12	0.421	0.675
Control Group	30	14.53	3.25		

Comparison of Pre-Test Scores Between Experimental and Control Groups

Prior to the CPA-based intervention, a pre-test monitored students' foundational numeracy skills. Results showed that the experimental and control groups had no statistically significant differences prior to the intervention and so had numeracy achievement levels that were the same at the start of the study.

Table 7: Post-Test Scores of Experimental and Control Groups

Group	N	Mean Score	Standard Deviation	t-value	p-value
Experimental Group	30	22.86	2.45	5.782	0.001
Control Group	30	16.73	3.10		

The pre-test scores showed that the groups were the same and so, the instructional approach, and not the differences in students' initial abilities, is what was responsible for the significant improvements that were observed post intervention.

Comparison of Post-Test Scores Between Experimental and Control Groups

A post-test was given to assess the numeracy achievement of students after the instructional intervention. The findings demonstrated that the participants in the experimental group performed better than the participants in the control group. The students who were taught using the CPA-based instruction method showed positive results with their understanding of early mathematical concepts.

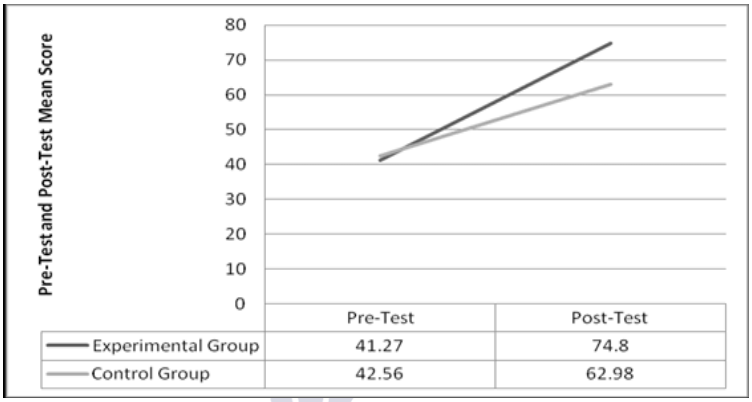

Fig 4.2 Comparison of Post-Test Scores Between Experimental and Control Groups

Table 8: Improvement in Numeracy Scores from Pre-Test to Post-Test

Group	Pre-Test Mean	Post-Test Mean	Mean Difference	Improvement Percentage
Experimental Group	14.26	22.86	8.60	60.3%
Control Group	14.53	16.73	2.20	15.1%

Students who participated in the CPA-based activities performed better with numbers, operations, reasoning, and even problem-solving compared to students who were taught using the conventional method.

Effect of CPA Approach on Foundational Numeracy Achievement

Research shows that using CPA-based instruction has a lasting impact on the development of foundational numeracy. Improvements in foundational numeracy for the experimental group were more significant compared with the control group. The process of transitioning from physical materials to drawings and finally to mathematical symbols of an abstract nature enables students to develop a better understanding.

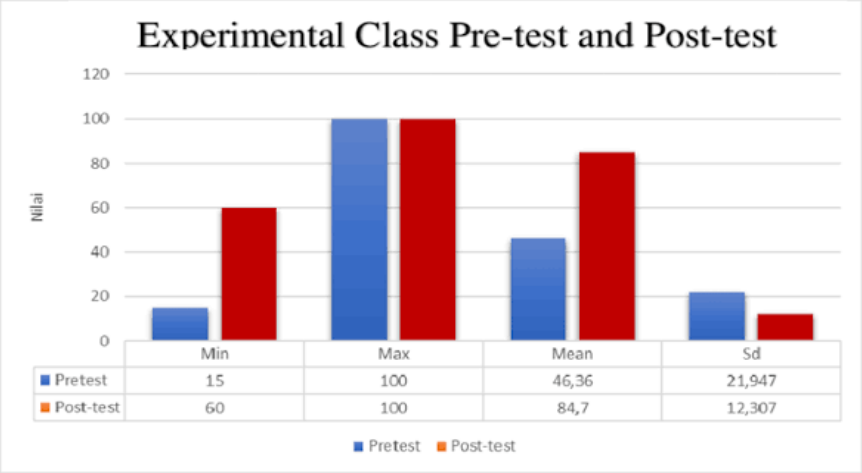

Fig 4.3 Effect of CPA Approach on Foundational Numeracy Achievement

These findings suggest that CPA allows students to have meaningful experiences that promote exploration of mathematical concepts and ultimately deepen that learning to enhance students' abilities to use mathematics.

Analysis of Students' Mathematical Reasoning and Problem-Solving Skills

The analysis showed that students who took part in CPA lessons showed greater advancement in both mathematical reasoning and problem solving. The use of concrete and representational resources helped students recognize the relationships between numbers and operations of mathematics and better understand them.

Students in the experimental group were also better able to verbalize their mathematical thinking, recognize a solution strategy, and transfer the same concept to different problems. Thus, APA instruction enhances not only the basic skills of mathematics but also the higher order skills of mathematics.

DISCUSSION

This research indicates that the Concrete-Pictorial-Abstract (CPA) method shows tremendous promise for developing primary learners' basic numeracy skills. Pupils who underwent CPA-based instruction had higher levels of attainment in numeracy when contrasted with their peers who did not receive CPA-based instruction. The findings have the implication that learners who have more of the opportunity to manipulate mathematical ideas using physical objects, as well as drawings and mathematical symbols, have a more developed understanding of the ideas of mathematics and how to apply them. The findings are consistent with prior studies that show the value in the use of combined instructional representations of ideas of mathematics.

One of the reasons for the positive change in the experimental group was the CPA method's ability to break down learning into manageable steps. Instead of projecting symbols of mathematics, the CPA method slowly guides the learners through physical models, drawings, and then mathematical symbols. This kind of gradual approach reduces the burden on learners' cognitive resources.

Impact of CPA Approach on Students' Numeracy Skills

The data are clear. The CPA method positively affects students' basic numeracy skills. Skills such as numeracy, calculation, and logic. By interpreting the CPA method's use of physical representations, students were able to understand the relationships of numbers before executing formal mathematical operations. This finding supports Carbonneau et al. (2018) who identified that concrete manipulatives have a place in mathematics learning, provided they are engaged with proper teaching strategies.

Relationship Between CPA Framework and Mathematical Understanding

The CPA approach successfully addresses the demands of a constructivist learning theory and a theory of cognitive development. Learners construct knowledge when they interact with learning materialsand when they engage in purposeful learning and mathematical activities. The CPA framework, at a given time, assists learners in developing their own understanding of mathematics, rather than the memorization of procedures.

The findings also reflect the tenets of Bruner's representation theory, which claims that learners gradually shift their understanding from the enactive, to the iconic, and then to the symbolic. The CPA framework, through its concrete, pictorial, and abstract stages, is designed to assist learners from the informal to the formal realm of mathematical thinking and reasoning. The structure provided through the CPA framework positively develops the learners' confidence and autonomy for the resolution of mathematical tasks.

Comparison with Previous Research Findings

This research aligns with earlier studies confirming the efficacy of CPA-based mathematics teaching. According to Putri et al. (2020), students who learned using the CPA approach showed a considerable advancement in mathematical reasoning compared to students learning through the traditional way. Also, CPA instruction, as per Lai and Yeo (2021), enhanced students' problem-solving ability and increased engagement with mathematics.

This research adds to the aforementioned studies by addressing the context of Pakistani primary education, where very little research on CPA is available. Although international research has recognized the advantages of CPA, this study aims to show its likely use in teaching situations where students struggle to deal with the abstractness of mathematical ideas.

Implications for Primary Mathematics Education in Pakistan

The evidence of this study has multiple ramifications on the teaching of mathematics in Pakistan. CPA allows the teacher to organize the class in a way that learning becomes interactive, engaging, and constructive. Teachers are encouraged to use CPA-based strategies rather than relying on rote learning and a textbook, and are encouraged to use manipulatives and visual models to support learning.

For policymakers and curriculum designers, integrating CPA-based learning strategies in the primary mathematics curriculum and developing teacher training programs to deepen teachers' understanding of CPA is critical. This focus on CPA would likely improve primary mathematics teaching and learning in Pakistan.

The study shows that the Concrete–Pictorial–Abstract approach is very useful in the support of early mathematics learning. The development of different concepts, the different types of representations, and the level of engagement that this approach includes makes it effective in building the first ideas of numeracy of learners.

CONCLUSION AND RECOMMENDATIONS

This research focused on the success and challenges of the Concrete–Pictorial–Abstract (CPA) approach for primary school children and the development of basic numeracy skills in the Pakistani context. It was found that students receiving myriad CPA instruction demonstrated significant gains in numeracy achievement as compared to students receiving instruction through the traditional methods. It was also noted that the CPA approach offered students a meaningful and organized pathway of learning and helped them progressively transition from hands-on learning with concrete tools to pictorial and abstract concepts of mathematics.

This research also showed that an important consideration for the development of basic numeracy skills is the adoption of teaching approaches that emphasize the active participation of students in the learning process,

along with the development of higher-order thinking skills. Approaches and teaching methods that emphasize rote learning and the learning of algorithms do not help students develop a deeper understanding of mathematics. CPA instruction, on the other hand, enables learners to make meaningful relationships among various mathematical concepts, which in turn enhances their problem-solving skills and confidence toward the learning of mathematics.

This study also provides useful insights for the integration of CPA instruction in the primary level of the Pakistani education system. While other studies have documented CPA's international success, this study expands the existing literature for the improvement of early mathematics instruction in developing educational contexts. This study also emphasizes that the incorporation of concrete, pictorial, and abstract methods of mathematics will enhance the basic numeracy skills of the students.

Educational Implications

The results of the study have predominant effects on teaching mathematics and educational planning. It is foremost important to advance CPA-based activities from the primary school teachers to formulate an engaging and interactive class that is rich in concepts and focuses on teaching. CPA Strategies can be included in pre-service training of teachers to prepare them to use teaching aids, drawings, and engage in activities that provoke thoughts and answers.

Curriculum designers and educational authorities should include activities that promote CPA in primary mathematics. Providing teachers with resources, training, and support to make activities based on CPA offers the best probability to achieve the goal. Designing activities from CPA Strategies will improve the teaching and learning of mathematics and tackle the gaps of basic numeracy in Pakistani schools.

Recommendations for Teachers and Schools

From the study findings, it is recommended that;

- Primary mathematics teachers should use concrete resources and visual aids on a daily basis.
- Schools should have adequate instructional materials and mathematical manipulatives.
- Focus should be on developing the conceptual understanding of students rather than rote learning.
- Teach CPA-based mathematics using appropriate techniques acquired from the courses.
- Organize activities which motivate students to verbalize their mathematical thinking.

Recommendations for Future Research

Subsequent studies can evaluate the CPA approach for effectiveness at different grades and across a range of educational contexts in Pakistan. The impact of CPA-based instruction on the achievement of mathematics over time and retention of the knowledge can also be researched.

Future studies could focus on the perceptions and experiences of primary school teachers regarding the challenges of implementing CPA. Longitudinal mixed research could examine the influence of CPA on students and the development of their mathematical learning behaviors and attitudes over a period of time. It might also be useful to consider research that compares CPA with other contemporary strategies for teaching mathematics.

REFERENCES

Azzumar, F., & Juandi, D. (2023). A Systematic Literature Review of the Concrete-Pictorial-Abstract Approach in Mathematics Education. Journal of Mathematics Education, 8(2), 115–130.

Bruner, J. S. (2018). The Process of Education: Updated Perspectives on Cognitive Learning and Representation. Harvard University Press.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2018). A Meta-analysis of the Efficacy of Teaching Mathematics with Concrete Manipulatives. Journal of Educational Psychology, 110(4), 545–562.

Clements, D. H., & Sarama, J. (2018). Learning and Teaching Early Math: The Learning Trajectories Approach. Routledge.

Clements, D. H., Sarama, J., & Germeroth, C. (2019). Learning and Teaching Early Mathematics through Learning Trajectories. Educational Researcher, 48(6), 353–365.

Duncan, G. J., & Magnuson, K. (2018). The Nature and Impact of Early Mathematics Skills on Later Achievement. Annual Review of Psychology, 69, 245–270.

Fyfe, E. R., McNeil, N. M., Son, J. Y., & Goldstone, R. L. (2019). Concreteness Fading in Mathematics Learning: Supporting Transfer and Abstraction. Educational Psychology Review, 31, 541–567.

Griffiths, R., Back, J., Gifford, S., & Watson, A. (2020). Developing Mathematical Understanding through Early Years Learning Experiences. Mathematics Education Research Journal, 32, 395–412.

Kaur, B. (2019). Revisiting Bruner's Framework: The Role of Representation in Mathematics Learning. Mathematics Education Research Journal,

Concrete–Pictorial–Abstract Approach and Mathematical Problem Solving

Lai and Yeo (2021) explored how the Concrete–Pictorial–Abstract approach impacts one's mathematical problem-solving abilities.

Representations and Constructs in Learning Mathematics

Lesh, Post, and Behr (2018) presented a study on the different models and their representations that aid in the learning of mathematics.

Pakistan's Foundational Learning National Education Policy

The Ministry of Federal Education and Professional Training (2020) emphasized improving the foundational learning structures in the education of the citizens of Pakistan within the policy framework of the government.

Technological Learning and Mathematical Representations

Moyer-Packenham, Bolyard, and Oh (2021) studied the role of representation and technology in learning elementary mathematics.

Mullis et al. (2020) provided the results of the 2019 assessment of the Trends in International Mathematics and Science Study.

The National Council of Teachers of Mathematics (2020) provided a resource for changing early childhood and elementary mathematics.

PISA 2018: Assessment and Analytical Framework

OECD (2019) published the assessment and analytical framework for the PISA 2018, covering mathematics, reading, and science literacy.

Putri et al. (2020) investigated the Concrete–Pictorial–Abstract approach and mathematical reasoning skills for elementary school students.

Early Childhood Mathematics Education Research

Sarama and Clements (2020) focused on learning research and teaching early childhood mathematics.

Mathematical Problem Solving and Understanding

Schoenfeld (2019) studied problem-solving and understanding mathematics in classroom settings.

Lastly, Siegler and Ramani (2018) conducted a study on early numerical knowledge and children's development in mathematics.

Sweller, J., Ayres, P., & Kalyuga, S. (2019). Cognitive Load Theory and Educational Technology. Springer.

UNESCO. (2019). Behind the Numbers: Ending School Violence and Bullying. UNESCO Publishing.

UNICEF. (2021). Reimagining Human Connections: Converging Education, Skills and Employment. UNICEF.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2019). Elementary and Middle School Mathematics: Teaching Developmentally. Pearson.

Wang, J., & Hsieh, T. (2020). The role of visual representation in improving students' mathematical reasoning. Educational Studies in Mathematics, 104, 257–275.

World Bank. (2022). The State of Global Learning Poverty: Update and Implications for Education Systems. World Bank Publications.

Zhang, X., Clements, D. H., & Sarama, J. (2021). Early mathematics learning through structured representations and instructional support. Early Childhood Research Quarterly, 54, 192–205.

Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2018). The relationship between mathematical understanding and representations. Mathematics Education Research Journal, 30, 415–432.

Bofferding, L. (2019). Developing children's number sense through conceptual mathematical activities. Journal of Mathematical Behavior, 54, 100–112.

Boaler, J. (2018). Mathematical Mindsets: Unleashing Students' Potential Through Creative Mathematics. Jossey-Bass.

Ding, M., & Li, X. (2020). Teaching mathematics through multiple representations: Evidence from primary classrooms. Educational Studies in Mathematics, 105, 145–164.

Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2019). Mathematics education for young children: What it is and how to promote it. Early Education and Development, 30, 1–15.

IEER
Institute for Excellence in Education & Research